# Nonlinear Random Response of Angle-Ply Laminates with Static and Thermal Preloads

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**This paper presents the results of an analytical study conducted to investigate the large-deflection nonlinear response of rectangular antisymmetric angle-ply laminated composite plates subjected to combinations of static, thermal, and acoustic loads. The analytical formulation used herein is based on the classical Karman-type strain nonlinearity and includes the effect of initial deflection, initial stress, and temperature. Variations in loading conditions are studied for different ply configurations to determine the effect on root-mean-square deflection and strain. This research should serve as a guide for the sonic fatigue design of angle-ply laminates in thermal/acoustic environments, lead to the development of improved analytical design methods for such structures, and aid in understanding their fundamental behavior.**

## Nomenclature

$a, b$ = plate length and width
$A, B, D$ = laminate stiffnesses
$e$ = membrane strain
$E[\ ]$ = expected value
$E_1, E_2$ = Young's moduli in longitudinal and transverse material directions
$F$ = stress function
$G_{12}$ = shear modulus
$h$ = plate thickness
$k$ = equivalent linear stiffness
$m$ = mass coefficient
$M$ = moment resultant
$n$ = number of layers
$N$ = in-plane force resultant
$p_0$ = static pressure
$p(t)$ = acoustic pressure
$q$ = modal amplitude
$r$ = length-to-width ratio $a/b$
$S$ = nondimensional spectral density parameter, Eq. (22)
$S_0$ = acoustic pressure spectral density
$t$ = time
$T$ = change in temperature
$T_{cr}$ = plate buckling temperature
$u, v, w$ = displacements
$x, y, z$ = coordinates
$\beta$ = nonlinear stiffness coefficient
$\varepsilon$ = total strain
$\eta$ = stiffness reduction coefficient, Eq. (16)
$\kappa$ = curvature
$\upsilon_{12}$ = Poisson's ratio
$\omega_0$ = linear radial frequency
$\phi$ = deflection function
$\rho$ = mass density
$\theta$ = lamination angle
$\zeta$ = damping ratio

*Subscripts*

$c$ = complementary
$p$ = particular
1 = first-order
2 = second-order

*Superscripts*

0 = initial
$T$ = thermal

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## Introduction

WITH the renewed interest in hypersonic and high-speed vehicles, it has become increasingly important to improve analytical methods used for sonic fatigue design. Improving current methods and developing new methods for these applications presents a challenge, since elevated temperatures, high noise levels, and composite materials must be considered.[1] Example applications include the National Aerospace Plane as well as high-speed civil transport aircraft.

Past studies[2-6] have shown that when thin structural panels are subjected to high acoustic loads, the response tends to be nonlinear. Furthermore, small changes in temperature and/or initial deflection have been shown[3-6] to have a pronounced effect on the panel response and strains. Of particular interest is the observation that the strain response of buckled and initially curved panels differs significantly from the deflection response. For a given acoustic loading, the root-mean-square (rms) deflection decreases with increasing initial deflection. The rms strain response, on the other hand, can either increase or decrease with increasing initial deflection, depending on the level of acoustic loading. This type of behavior is due to strain coupling between the initial deflection (due to mechanical or thermal loads) and the random deflection due to the acoustic loading.

Current analytical design methods for sonic fatigue are based primarily on linear theory and are incapable of accounting for the above-mentioned nonlinearities. The inclusion of nonlinear effects is essential if static and thermal preloads are to be considered. For composite materials the analysis is further complicated because, in general, bending and extensional deformations can be fully coupled. A review of the literature reveals that no previous analytical studies have considered the nonlinear random response of laminated composite plates with static and thermal preloads. The purpose of the present study is to develop a mathematical model for such structures and to use this model to investigate the effect of static and thermal preloads on the rms response and strains. The types of plates considered are rectangular antisymmetric angle-ply laminates with all edges clamped and all edges simply supported. Specially orthotropic laminates can be treated as a special case. Both movable and immovable in-plane edges are considered.

For the present study, the classical Kirchhoff thin-plate theory is used in conjunction with the Karman-type strain nonlinearity, and the equations of motion are expressed in terms of a stress function and the out-of-plane deflection. All of the loads (static, thermal, and acoustic) are assumed to be uniformly applied, and the acoustic loading is assumed to be stationary white noise. A single-mode Galerkin approach is used to reduce the coupled system of nonlinear partial differential equations to a single, time-dependent, ordinary, nonlinear differential equation. An iterative procedure is used to solve for the initial deflection due to the static and thermal preloads. For the acoustic loading, the method of equivalent linearization[7] is used to determine the rms response.

## Equations of Motion

The von Karman strain-displacement relations, modified to include the effect of a large initial deflection, $w^0$, are given by

$$\{\varepsilon\} = \{e\} + z\{\kappa\} \tag{1}$$

where

$$\{e\} = \begin{Bmatrix} u_{,x} + \frac{1}{2}w_{,x}^2 + w_{,x}w_{,x}^0 \\ v_{,y} + \frac{1}{2}w_{,y}^2 + w_{,y}w_{,y}^0 \\ u_{,y} + v_{,x} + w_{,x}w_{,y} + w_{,x}w_{,y}^0 + w_{,y}w_{,x}^0 \end{Bmatrix}$$

$$\{\kappa\} = -\begin{Bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{Bmatrix}$$

and the deflection $w$ is measured with respect to the initial deflection $w^0$. Note that these strains are due to a displacement away from the initial equilibrium configuration $w^0$. Using these strain-displacement equations in conjunction with classical lamination theory leads to the constitutive equations[8]

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} e \\ \kappa \end{Bmatrix} + \begin{Bmatrix} N^0 - N^T \\ M^0 - M^T \end{Bmatrix} \tag{2}$$

where $N^0$, $M^0$ are due to initial stresses and $N^T$, $M^T$ are due to a change in temperature. Applying the principle of virtual work,

$$\delta W = \delta W_{\text{int}} - \delta W_{\text{ext}} = 0 \tag{3}$$

where

$$\delta W_{\text{int}} = \int_A (\{\delta e\}^T\{N\} + \{\delta\kappa\}^T\{M\})\, dA$$

$$\delta W_{\text{ext}} = \int_A \delta w[p_0 + p(t) - \rho h\ddot{w}]\, dA$$

in conjunction with Eq. (1) leads to the following equations of motion:

$$N_{x,x} + N_{xy,y} = 0 \qquad N_{xy,x} + N_{y,y} = 0 \tag{4}$$

$$\begin{aligned} &M_{x,xx} + 2M_{xy,xy} + M_{y,yy} + N_x(w_{,xx} + w_{,xx}^0) \\ &+ 2N_{xy}(w_{,xy} + w_{,xy}^0) + N_y(w_{,yy} + w_{,yy}^0) \\ &+ p_0 + p(t) = \rho h\ddot{w} \end{aligned} \tag{5}$$

To express these equations of motion in terms of the out-of-plane deflection and a stress function, Eq. (2) is written in partially inverted form as

$$\begin{Bmatrix} e \\ \overline{M} \end{Bmatrix} = \begin{bmatrix} A^* & B^* \\ -(B^*)^T & D^* \end{bmatrix} \begin{Bmatrix} \overline{N} \\ \kappa \end{Bmatrix} \tag{6}$$

where

$$A^* = A^{-1} \qquad B^* = -A^*B \qquad D^* = D + BB^*$$
$$\overline{M} = M + M^T - M^0 \qquad \overline{N} = N + N^T - N^0$$

and the in-plane forces are defined in terms of the Airy stress function as

$$N = \begin{Bmatrix} F_{,yy} \\ F_{,xx} \\ -F_{,xy} \end{Bmatrix} \qquad N^0 = \begin{Bmatrix} F_{,yy}^0 \\ F_{,xx}^0 \\ -F_{,xy}^0 \end{Bmatrix} \tag{7}$$

For an antisymmetric angle-ply laminate, $A_{16} = A_{26} = B_{11} = B_{12} = B_{22} = D_{16} = D_{26} = 0$. Using these values in conjunction with Eqs. (1) and (5–7) yields the equation of motion for the out-of-plane deflection $w$:

$$\begin{aligned} &\rho h\ddot{w} + L_1 w + L_3\overline{F} - L_4(F,w) \\ &- L_4(\overline{F},w^0) - p(t) = 0 \end{aligned} \tag{8}$$

where

$$L_1 = D_{11}^* \frac{\partial^4}{\partial x^4} + 2(D_{12}^* + 2D_{66}^*)\frac{\partial^4}{\partial x^2\partial y^2} + D_{22}^*\frac{\partial^4}{\partial y^4}$$

$$L_3 = (2B_{26}^* - B_{61}^*)\frac{\partial^4}{\partial x^3\partial y} + (2B_{16}^* - B_{62}^*)\frac{\partial^4}{\partial x\partial y^3}$$

$$L_4(F,w) = F_{,yy}w_{,xx} + F_{,xx}w_{,yy} - 2F_{,xy}w_{,xy}$$

$$\overline{F} = F - F^0$$

and the terms $F^0$, $w^0$ correspond to the in-plane forces and the out-of-plane deflection produced by static preloads $p_0$ (a uniformly applied pressure) and $T$ (a uniform change in temperature). That is, $p_0$ and $T$ are applied to the initially flat, stress-free plate to produce $F^0$ and $w^0$.

The compatibility equation for a plate with initial deflections [derived from Eq. (1)] is

$$\begin{aligned} &e_{x,yy} + e_{y,xx} - e_{xy,xy} \\ &+ \tfrac{1}{2}L_4(w,w) + L_4(w,w^0) = 0 \end{aligned} \tag{9}$$

Making use of Eqs. (6) and (7), the compatibility equation can be rewritten in terms of the stress function and the out-of-plane deflection as

$$L_2\overline{F} - L_3 w + \tfrac{1}{2}L_4(w,w) + L_4(w,w^0) = 0 \tag{10}$$

where

$$L_2 = A_{22}^*\frac{\partial^4}{\partial x^4} + (2A_{12}^* + A_{66}^*)\frac{\partial^4}{\partial x^2\partial y^2} + A_{11}^*\frac{\partial^4}{\partial y^4}$$

## Galerkin Formulation

An approximate solution for Eq. (8) can be obtained by assuming that the out-of-plane deflections are given by

$$w = q(t)h\phi(x,y) \qquad w^0 = q^0 h\phi(x,y) \tag{11}$$

where $\phi(x,y)$ satisfies the plate boundary conditions, $q(t)$ is a time-dependent modal amplitude due to acoustic pressure $p(t)$, and $q^0$ is known.

The stress function is assumed to be of the form

$$F = F_c + F_p \tag{12}$$

Substitution of Eqs. (11) into Eq. (10) results in the following general expressions for the particular solution:

$$\begin{aligned} \bar{F}_p &= F_{p1}qh + F_{p2}[(q + q^0)^2 - q^{02}]h^2 \\ F_p &= F_{p1}(q + q^0)h + F_{p2}(q + q^0)^2h^2 \end{aligned} \tag{13}$$

The complementary solution is assumed as

$$F_c = P_y(x^2/2) + P_x(y^2/2) + P_{xy}xy$$

where the constants $P_x$, $P_y$, and $P_{xy}$ are obtained so that the in-plane boundary conditions are satisfied. For movable in-plane edges, the boundary conditions are

$$x = \pm a/2: \quad F_{,xy} = 0, \quad \int_{-b/2}^{b/2} F_{,yy}\, dy = 0$$

$$y = \pm b/2: \quad F_{,xy} = 0, \quad \int_{-a/2}^{a/2} F_{,xx}\, dx = 0$$

Applying the above boundary conditions, the complementary solution $F_c$ for movable in-plane edges is zero; thus, $F = F_p$.

For immovable in-plane edges, the boundary conditions are

$$x = \pm a/2: \quad F_{,xy} = 0, \quad \iint \left(e_x - \frac{1}{2} w_{,x}^2 - w_{,x}w_{,x}^0\right) dx\, dy = 0$$

$$y = \pm b/2: \quad F_{,xy} = 0, \quad \iint \left(e_y - \frac{1}{2} w_{,y}^2 - w_{,y}w_{,y}^0\right) dx\, dy = 0$$

Application of these boundary conditions yields the following general expressions for the complementary solution:

$$\begin{aligned} \bar{F}_c &= F_{c2}[(q + q^0)^2 - q^{02}]h^2 \\ F_c &= F_{c2}(q + q^0)^2h^2 - N_y^T(x^2/2) - N_x^T(y^2/2) \end{aligned} \tag{14}$$

Substitution of Eqs. (11), (13), and (14) into Eq. (8) and application of Galerkin's method yields the nonlinear equation of motion:

$$\ddot{q} + \omega_0^2(1 - \eta)q + \beta[(q + q^0)^3 - q^{03}] = p(t)/m \tag{15}$$

Equation (15) can be expressed in nondimensional form as

$$\ddot{q} + c\lambda_0^2(1 - \eta)q + c\beta^*[(q + q^0)^3 - q^{03}] = p(t)/m \tag{16}$$

where

$$c = E_2h^2/\rho b^4$$

$$\omega_0^2 = c\lambda_0^2$$

$$\beta = c\beta^*$$

$\eta = 0$ for movable in-plane edges

$\eta = T/T_{cr}$ for immovable in-plane edges

Expressions for the linear radial frequency parameter $\lambda_0$; the nonlinear stiffness coefficient $\beta^*$; the modal mass $m$; the stress function coefficients $F_{p1}$, $F_{p2}$, and $F_{c2}$; and the deflection function $\phi(x,y)$ are given in the Appendix for plates with all edges simply supported and all edges clamped.

## Method of Solution

To determine the random response $q(t)$ using Eq. (16), the initial static response $q^0$ must be known. Thus, the solution process actually involves two steps: 1) determine the initial static response $q^0$ due to static preloads $p_0$ and $T$, and 2) determine the random response $q(t)$ due to acoustic pressure $p(t)$.

### Static Response

Modifying Eq. (16) for an initially flat, stress-free plate subjected to static loads $p_0$ and $T$, the nonlinear modal equation to determine the static response $q^0$ becomes

$$\lambda_0^2(1 - \eta)q^0 + \beta^*q^{03} = P_0/m_f \tag{17}$$

where

$$P_0 = p_0b^4/E_2h^4$$

$m_f = \Pi^2/16$ for simply supported plate

$= 9/16$ for clamped plate

### Random Response

The nonlinear modal equation [Eq. (15)], modified to include the effect of linear modal damping, takes the form

$$\ddot{q} + 2\zeta\omega_0\dot{q} + \omega_0^2(1 - \eta)q + \beta[(q + q^0)^3 - q^{03}] = p(t)/m \tag{18}$$

where $\zeta$ is the damping ratio. The method of equivalent linearization can be used to obtain an approximate solution for Eq. (18). Consider that Eq. (18) can be written in the form

$$\ddot{q} + 2\zeta\omega_0\dot{q} + g(q) = p(t)/m \tag{19}$$

where

$$g(q) = \omega_0^2(1 - \eta)q + \beta[(q + q^0)^3 - q^{03}]$$

An equivalent linearized form of Eq. (19) can be written as

$$\ddot{q} + 2\zeta\omega^0\dot{q} + kq = p(t)/m \tag{20}$$

where $k$ is the equivalent linear stiffness. For a stationary white excitation $p(t)$ with uniform power spectral density $S_0$, the solution of Eq. (20) for the mean-square response of the modal amplitude is given by[9]

$$E[q^2] = \frac{S_0}{8m^2\zeta\omega_0k} \tag{21}$$

where $S_0$ is single sided with units of (pressure)$^2$/Hz. Equation (21) can be expressed in nondimensional form as

$$E[q^2] = S/8m_f^2\zeta\lambda_0k^* \tag{22}$$

where

$$k^* = k/(E_2h^2/\rho b^4)$$

$$S = S_0/[\rho^2h^4(E_2h^2/\rho b^4)^{3/2}$$

The error involved in using Eq. (20) instead of Eq. (19) is given by the difference between the two equations as

$$\text{error} = g(q) - kq \tag{23}$$

The method of equivalent linearization requires that the mean square error be a minimum with respect to the equivalent linear stiffness $k$; that is

$$\frac{\partial}{\partial k} E[\text{error}^2] = 0 \tag{24}$$

Using Eqs. (19), (23), and (24), the equivalent linear stiffness $k$ is found to be

$$k = \omega_0^2 (1 - \eta) + 3\beta E[q^2] + 3\beta q^{02} \tag{25}$$

which can be expressed in nondimensional form as

$$k^* = \lambda_0^2 (1 - \eta) + 3\beta^* E[q^2] + 3\beta^* q^{02} \tag{26}$$

Substituting Eq. (26) into Eq. (22), the mean-square response of the modal amplitude is found to be

$$E[q^2] = (-B + \sqrt{B^2 + 4AC})/2A \tag{27}$$

where

$$A = 3\beta^*$$
$$B = \lambda_0^2 (1 - \eta) + 3\beta^* q^{02}$$
$$C = S/8m_f^2 \zeta \lambda_0$$

After the rms displacement is determined, the strains can be evaluated using Eqs. (1) and (6). For the present study, only the $x$ component of the surface strain ($z = h/2$) is evaluated. The general form can be expressed as

$$(b/h)^2 \varepsilon_x = D_1 q + D_2[(q + q^0)^2 - q^{02}] \tag{28}$$

The constants $D_1$ and $D_2$ are given in the Appendix for a plate with all edges simply supported. Of course, similar expressions would also apply to the other strain components and to plates with all edges clamped. Using Eq. (28), the mean-square strain can be expressed in terms of the mean-square modal response as

$$(b/h)^4 E[\varepsilon_x^2] = (D_1 + 2q^0 D_2)^2 E[q^2] + 3D_2^2 (E[q^2])^2 \tag{29}$$

**Summary of Response Computations**

Given: static preloads $p_0$ and $T$
acoustic pressure loading $p(t)$ with power spectral density $S_0$

1) Compute initial static response $q^0$, Eq. (17)
2) Compute mean-square response $E[q^2]$, Eq. (27)
3) Compute mean-square strain $E[\varepsilon_x^2]$, Eq. (29)

## Results and Discussion

The primary objective of this study is to investigate the effect of static and thermal preloads on the random displacement and strain response of laminated composite plates. Both simply supported and clamped square plates with movable and immovable in-plane edges are considered. A 15- × 15- × 0.040-in. regular antisymmetric angle-ply ($\theta/-\theta/\theta/-\theta/\ldots$, $n$ = even, $h_i = h/n$) laminated plate with properties

$$E_1 = 30 \times 10^6 \text{ psi}$$
$$E_2 = 0.75 \times 10^6 \text{ psi}$$
$$G_{12} = 0.375 \times 10^6 \text{ psi}$$
$$\upsilon_{12} = 0.25$$
$$\rho = 2.4 \times 10^{-4} \text{ lb-s}^2/\text{in.}^4$$

and a damping ratio of $\zeta = 0.01$ is used for all computations.

For a four-layer (30°/−30°/30°/−30°) laminate, the rms maximum deflection is given as a function of static, thermal, and acoustic loading in Figs. 1–5. The nondimensional acoustic loading levels of $S$ = 1, 100, 10,000 correspond to sound spectrum levels of 72.4, 92.4, and 112.4 dB, respectively.

Figures 1 and 2 illustrate the effect of nondimensional thermal and static preloads ($T/T_{cr}$ and $P_0$) on the rms response of plates with immovable in-plane edges. For both cases the rms response is maximum when the initial out-of-plane deflection $w^0$ is zero. As the initial deflection increases, the stiffness increases, and the rms response decreases. This type of behavior is consistent with behavior exhibited by isotropic plates and beams. As shown in Fig. 3, plates with movable in-plane edges behave in a similar fashion.

For movable in-plane edges, only a static preload is considered, since for a uniform thermal loading no out-of-plane deflection occurs when the plate is free to expand. Figures 4 and 5 compare the effect of the two preloads by showing the rms response as a function of the initial out-of-plane deflection and the acoustic loading. The initial out-of-plane deflection for these figures was computed using the ranges for the thermal and static preloads shown in Figs. 1 and 2, respectively. For both types of plates, clamped and simply supported, the

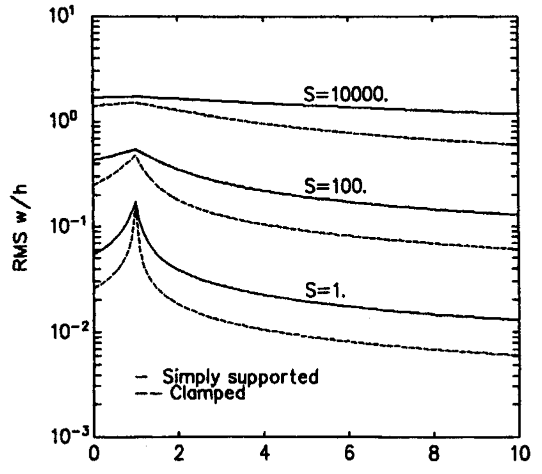


**Fig. 1 Effect of thermal preload on rms deflection of four-layer (30°/−30°/30°/−30°) square plate with immovable in-plane edges.**

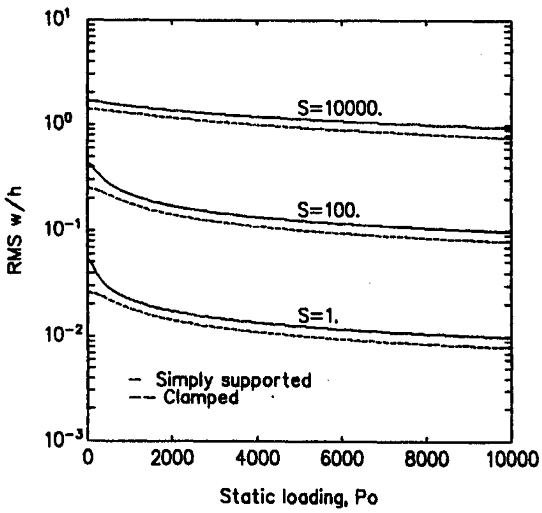


**Fig. 2 Effect of static preload on rms deflection of four-layer (30°/−30°/30°/−30°) square plate with immovable in-plane edges.**

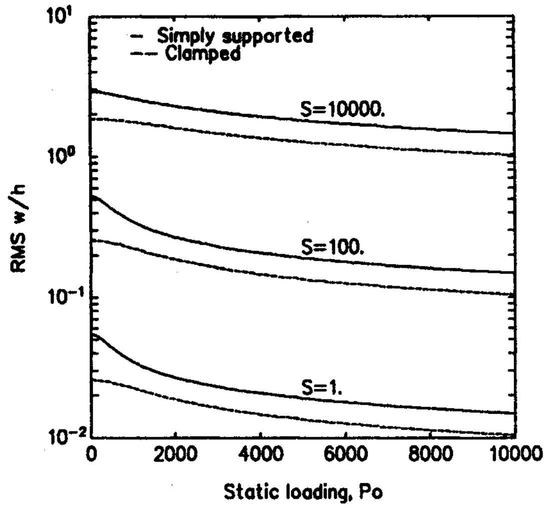


**Fig. 3 Effect of static preload on rms deflection of four-layer (30°/−30°/30°/−30°) square plate with movable in-plane edges.**

maximum rms response for a given initial deflection and acoustic pressure occurs for a thermal preload. This is due to the decrease in linear stiffness for a buckled plate. This, of course, might not be the case if the buckling mode shape differed from the vibration mode shape. For the present study, these two were assumed to be identical. The rms response for a combined static/thermal preload was found to fall between the response curves shown in Figs. 4 and 5.

To investigate different ply configurations, variations in lamination angle and number of layers are considered for two cases: plates with immovable in-plane edges subjected to a thermal preload, and plates with movable in-plane edges subjected to a static preload. For both cases a nondimensional acoustic loading of $S = 5000$ is used. Figure 6 shows that for a simply supported thermally buckled plate ($T/T_{cr} = 1$) the rms response tends to increase with lamination angle until a value of 30 deg for any number of layers. The orthotropic case corresponds to an infinite number of layers for which the terms $B_{16} = B_{26} = 0$. Note that the postbuckled plate ($T/T_{cr} = 5$) behavior significantly differs from the buckled plate behavior. For a two-layer laminate, the response is a maximum near 30 deg, but for the four-layer and orthotropic

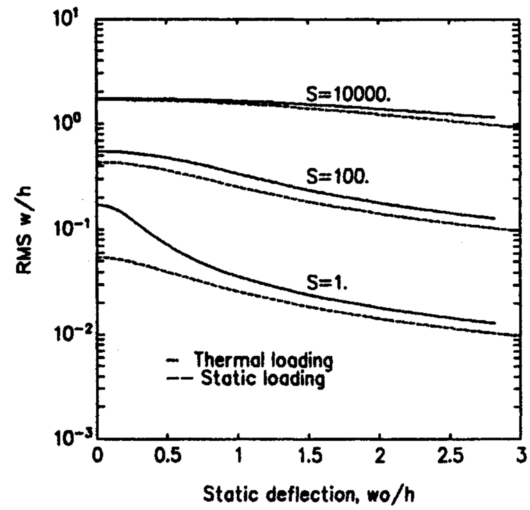


**Fig. 4 Effect of initial deflection on rms deflection of four-layer (30°/−30°/30°/−30°) simply supported square plate with immovable in-plane edges.**

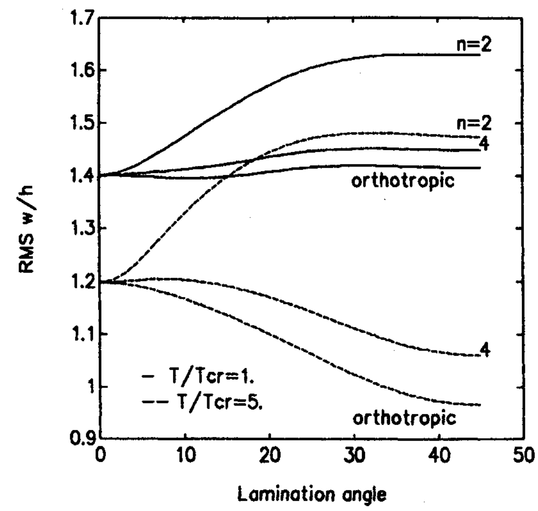


**Fig. 6 Rms deflection of simply supported square angle-ply plate with immovable in-plane edges and thermal preload at $S = 5000$.**

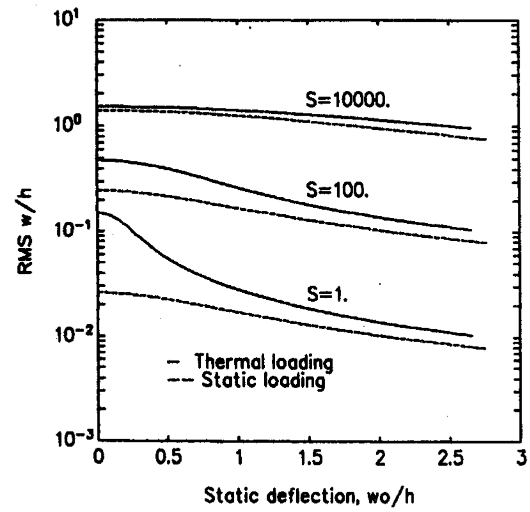


**Fig. 5 Effect of initial deflection on rms deflection of four-layer (30°/−30°/30°/−30°) clamped square plate with immovable in-plane edges.**

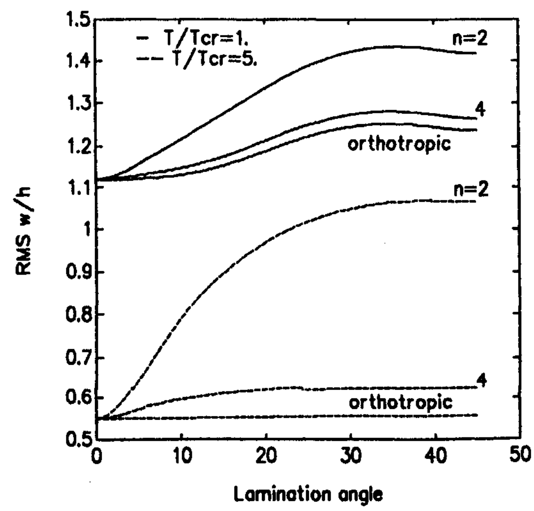


**Fig. 7 Rms deflection of clamped square angle-ply plate with immovable in-plane edges and thermal preload at $S = 5000$.**

laminates the response is maximum near 10 deg and 0 deg, respectively. Significant differences in the rms response also occur for a thermally loaded clamped plate with immovable in-plane edges, as shown in Fig. 7. Figures 8 and 9 show the effect of lamination angle and number of layers on the rms response of plates with movable in-plane edges subjected to a static preload. A preload of $P_0 = 0$ corresponds to a flat, stress-free plate. As demonstrated, the behavior for $P_0 = 0$ and $P_0 = 5000$ is very similar.

The rms maximum surface strain in the $x$-coordinate direction for a four-layer (30°/−30°/30°/−30°) simply supported square plate is shown in Figs. 10–12 for thermal and static preloads. An examination of Fig. 12 reveals that the strain response either increases or decreases with initial deflection depending on the level of acoustic loading. For low levels the rms strain decreases, but for higher levels it increases. This type of behavior is also consistent with that exhibited by isotropic plates and beams. The maximum rms strain for a plate with immovable in-plane edges occurs for the case of a thermal preload.

Variations in lamination angle and number of layers are considered for simply supported plates with immovable in-plane edges/thermal preload and movable in-plane edges/static preload. These two cases are shown in Figs. 13 and 14. As

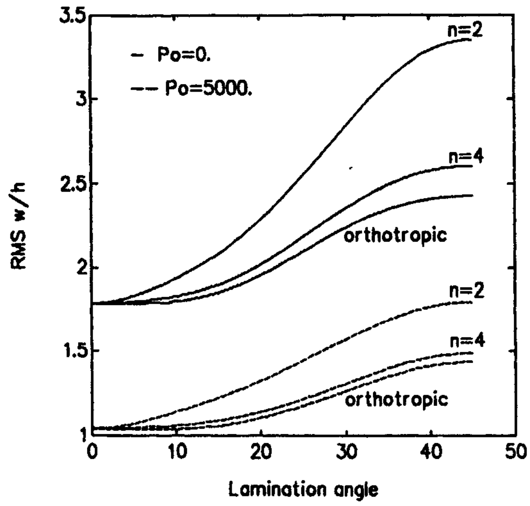


**Fig. 8 Rms deflection of simply supported square angle-ply plate with movable in-plane edges and thermal preload at $S = 5000$.**

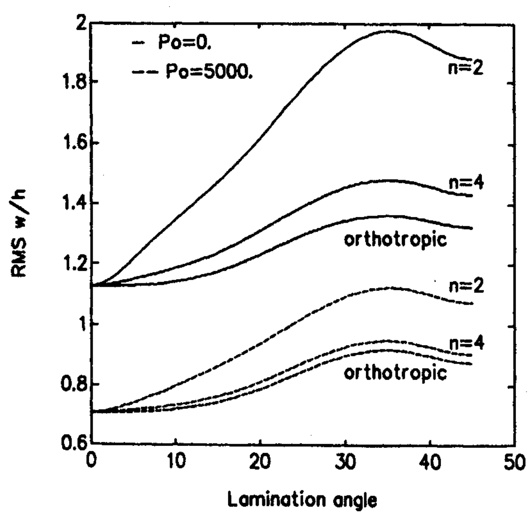


**Fig. 9 Rms deflection of clamped square angle-ply plate with movable in-plane edges and thermal preload at $S = 5000$.**

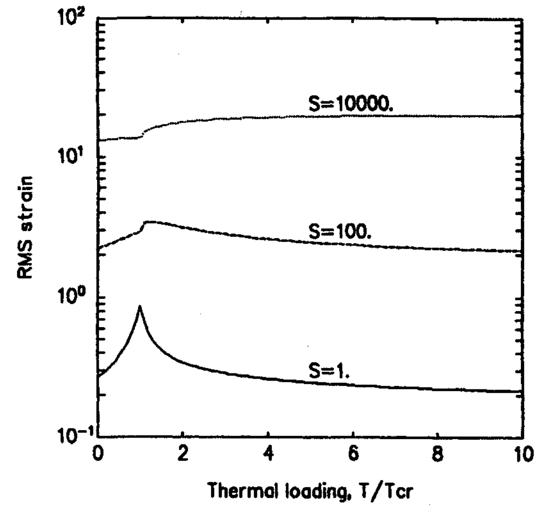


**Fig. 10 Effect of thermal preload on rms strain of four-layer (30°/−30°/30°/−30°) simply supported square plate with immovable in-plane edges.**

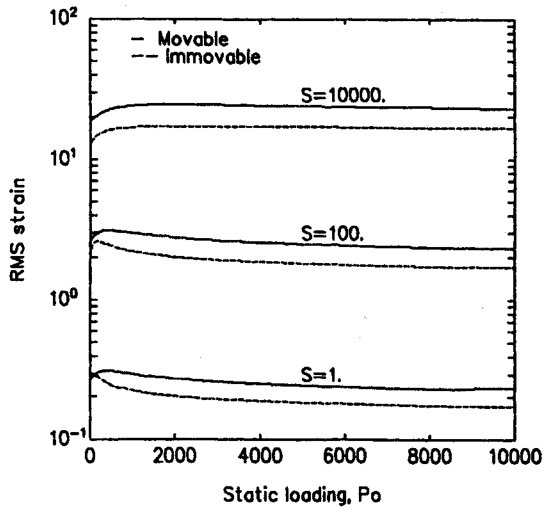


**Fig. 11 Effect of static preload on rms strain of four-layered (30°/−30°/30°/−30°) simply supported square plate.**

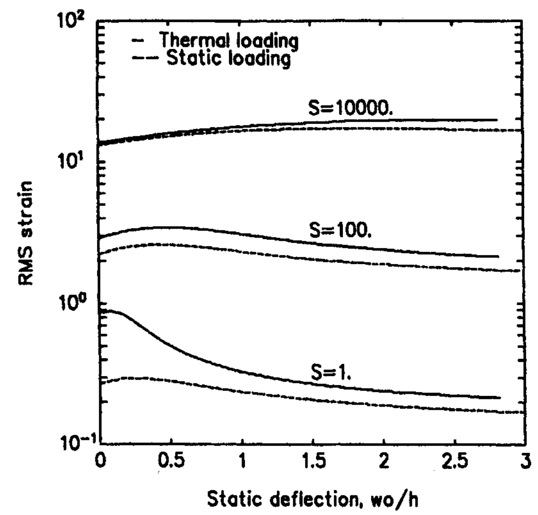


**Fig. 12 Effect of initial deflection on rms strain of four-layer (30°/−30°/30°/−30°) simply supported square plate with immovable in-plane edges.**

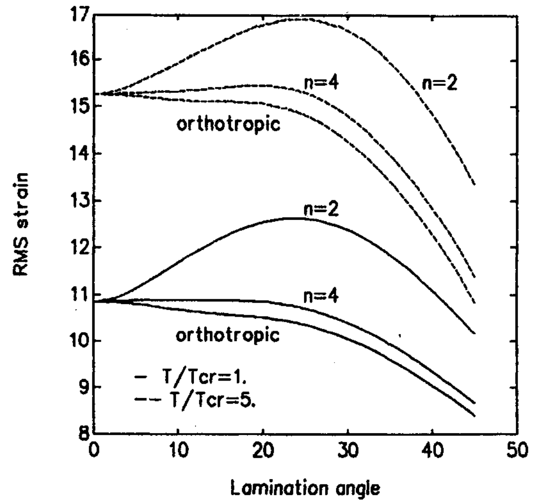


**Fig. 13 Rms strain of simply supported square angle-ply plate with immovable in-plane edges and thermal preload at $S = 5000$.**

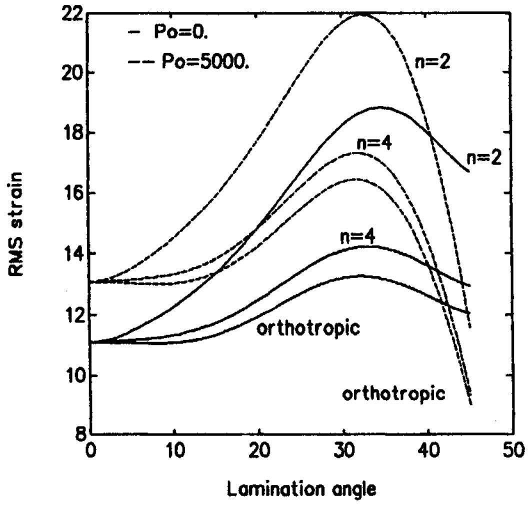


**Fig. 14 Rms strain of simply supported square angle-ply plate with movable in-plane edges and static preload at $S = 5000$.**

demonstrated earlier for the rms response, the effect of neglecting bending-extensional coupling is most significant for a two-layer laminate. Furthermore, the strain response can either increase or decrease with lamination angle depending on the type of loading and the edge conditions.

## Conclusions

An analytical method for determining the large deflection response and strain of antisymmetric angle-ply laminated composite plates subjected to static, thermal, and acoustic loads has been presented. A single-mode Galerkin solution was employed in conjunction with the method of equivalent linearization. The results demonstrate that the response and strain are highly dependent on the type of loading, the edge conditions, the ply lamination angle, and the number of layers. Three key assumptions have been made: 1) the equivalent linearization method is valid for initially deflected plates, 2) the initial deflection mode shape and vibration mode shape are identical, and 3) a single-mode solution is sufficient. Future studies should investigate the validity of these assumptions.

## Appendix

**Deflection Function**

*Simply Supported Plate:*

$$\phi(x,y) = \cos(\pi x/a)\cos(\pi y/b)$$

*Clamped Plate:*

$$= \tfrac{1}{4}[1 + \cos(2\pi x/a)][1 + \cos(2\pi y/b)]$$

**Stress Function Coefficients**

*Simply Supported Plate:*

$$F_{p1} = -F_{00}\sin(\pi x/a)\sin(\pi y/b)$$

$$F_{p2} = (-r^2/32)[F_{10}\cos(2\pi x/a) + F_{01}\cos(2\pi y/b)]$$

$$F_{c2} = \frac{\pi^2}{16(A^*_{11}A^*_{22} - A^{*2}_{12})}\left[\left(\frac{A^*_{11}}{b^2} - \frac{A^*_{12}}{a^2}\right)x^2 + \left(\frac{A^*_{22}}{a^2} - \frac{A^*_{12}}{b^2}\right)y^2\right]$$

*Clamped Plate:*

$$F_{p1} = -(F_{00}/4)\sin(2\pi x/a)\sin(2\pi y/b)$$

$$F_{p2} = (-r^2/32)[F_{10}\cos(2\pi x/a) + F_{01}\cos(2\pi y/b) + F_{11}\cos(2\pi x/a)\cos(2\pi y/b) + F_{20}\cos(4\pi x/a) + F_{02}\cos(4\pi y/b) + F_{21}\cos(4\pi x/a)\cos(2\pi y/b) + F_{12}\cos(2\pi x/a)\cos(4\pi y/b)]$$

$$F_{c2} = \frac{3\pi^2}{64(A^*_{11}A^*_{22} - A^{*2}_{12})}\left[\left(\frac{A^*_{11}}{b^2} - \frac{A^*_{12}}{a^2}\right)x^2 + \left(\frac{A^*_{22}}{a^2} - \frac{A^*_{12}}{b^2}\right)y^2\right]$$

where

$$F_{00} = \frac{(2B^*_{26} - B^*_{61})r + (2B^*_{16} - B^*_{62})r^3}{A^*_{22} + (2A^*_{12} + A^*_{66})r^2 + A^*_{11}r^4}$$

$$F_{10} = 1/A^*_{22} \qquad F_{01} = 1/A^*_{11}r^4$$

$$F_{11} = \frac{2}{A^*_{22} + (2A^*_{12} + A^*_{66})r^2 + A^*_{11}r^4}$$

$$F_{20} = 1/16A^*_{22} \qquad F_{02} = 1/16A^*_{11}r^4$$

$$F_{21} = \frac{1}{16A^*_{22} + 4(2A^*_{12} + A^*_{66})r^2 + A^*_{11}r^4}$$

$$F_{12} = \frac{1}{A^*_{22} + 4(2A^*_{12} + A^*_{66})r^2 + 16A^*_{11}r^4}$$

**Coefficients for Nonlinear Equation of Motion**

*Simply Supported Plate:*

$$m = \pi^2\rho h^2/16$$

$$\lambda_0^2 = (\pi^4/E_2h^3r^4)\{D^*_{11} + 2(D^*_{12} + 2D^*_{66})r^2 + D^*_{22}r^4 + F_{00}[(2B^*_{26} - B^*_{61})r + (2B^*_{16} - B^*_{62})r^3]\}$$

$$\beta^*_p = (\pi^4/16E_2h)(F_{10} + F_{01})$$

*Clamped Plate:*

$$m = 9\rho h^2/16$$

$$\lambda_0^2 = (16\pi^4/9E_2h^3r^4)\{3D_{11}^* + 2(D_{12}^* + 2D_{66}^*)r^2 + 3D_{22}^*r^4$$
$$+ F_{00}[(2B_{26}^* - B_{61}^*)r + (2B_{16}^* - B_{62}^*)r^3]\}$$

$$\beta_p^* = (\pi^4/9E_2h)[F_{10} + F_{01} + F_{11} + F_{20} + F_{02}$$
$$+ 1/2(F_{21} + F_{12})]$$

*Movable Edges:*

$$\beta^* = \beta_p^*$$

*Immovable Edges:*

$$\beta^* = \beta_p^* + \beta_c^*$$

where

$$\beta_c^* = \frac{\pi^4}{8E_2hr^4}\left(\frac{A_{22}^* - 2A_{12}^*r^2 + A_{11}^*r^4}{A_{11}^*A_{22}^* - A_{12}^{*2}}\right)$$

**Strain Coefficients for Simply Supported Plate**

$$D_1 = (\pi^2/r^2h)\{[F_{00}(A_{11}^*r^2 + A_{12}^*)$$
$$- 2B_{16}^*r]\sin(\pi x/a)\sin(\pi y/b)$$
$$+ (h/2)\cos(\pi x/a)\cos(\pi y/b)\}$$

$$D_{p2} = (\pi^2/8)[A_{12}^*F_{10}\cos(2\pi x/a) + A_{11}^*r^2F_{01}\cos(2\pi y/b)]$$

$$D_{c2} = \pi^2/8r^2$$

*Movable Edges:*

$$D_2 = D_{p2}$$

*Immovable Edges:*

$$D_2 = D_{p2} + D_{c2}$$

## Acknowledgment

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